

Let  $n = p_0^{a_0} p_1^{a_1} \cdots p_{m-1}^{a_{m-1}}$ .

Set

$$G = \prod_{i=0}^{m-1} \mathbb{Z}_{a_i+1}$$

with  $\rho : G \times \{p_i\}_{i=0}^{m-1} \rightarrow \mathbb{N}$  such that if  $g = (b_0, b_1, \dots, b_i, \dots, b_{m-1})$  then  $\rho(g, p_i) = b_i$  where  $b_i$  is the least nonnegative representative of  $b_i \in \mathbb{Z}_{a_i}$ .

Define

$$k = \frac{1}{2} \sum_{g \in G} \prod_{i=0}^{m-1} p_i^{\rho(g, p_i)}.$$

Then  $n$  is perfect if and only if  $n = k$ .