Fractions

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Abstract

This handout presents a method to add any fraction along with an explanation of the basic properties of addition and multiplication that make this work.

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

1. **Identify** a, b, c and d.
2. **Plug** in a, b, c and d into the second expression.
3. **Simplify** the second expression.

Example 1

$$\frac{3}{5}+\frac{7}{6}=?$$

1. **Identify** $a = 3$, $b = 5$, $c = 7$ and $d = 6$.

2. **Plug** in a, b, c and d into the second expression.

$$\frac{3}{5}+\frac{7}{6}=\frac{ad+bc}{bd}=\frac{(3)(6)+(5)(7)}{(5)(6)}.$$

3. **Simplify** the second expression.

$$\frac{3}{5}+\frac{7}{6}=\frac{(3)(6)+(5)(7)}{(5)(6)}=\frac{18+35}{30}=\frac{53}{30}=1\frac{13}{30}.$$

Example 2

$$\frac{2}{3}+\frac{2A}{9}=?$$

1. **Identify** $a = 2$, $b = 3$, $c = 2A$ and $d = 9$.

2. **Plug** in a, b, c and d into the second expression.

$$\frac{2}{3}+\frac{2A}{9}=\frac{ad+bc}{bd}=\frac{(2)(9)+(3)(2A)}{(3)(9)}.$$

3. **Simplify** the second expression.

$$\frac{2}{3}+\frac{2A}{9}=\frac{(18)+(6A)}{(27)}=\frac{(3)(2A+6)}{(3)(9)}=\frac{3}{3}\frac{2A+6}{9}=\frac{2A+6}{9}.$$

Example 3

$$\frac{6}{(iy)(iy-2)} + \frac{3}{iy} = ?$$

1. **Identify** $a = 6$, $b = iy(iy - 2)$, $c = 3$ and $d = iy$.
2. **Plug** in a, b, c and d into the second expression.

$$\frac{6}{(iy)(iy-2)} + \frac{3}{iy} = \frac{ad+bc}{bd} = \frac{(6)(iy) + ((iy)(iy-2))\,(3)}{((iy)(iy-2))\,(iy)}.$$

3. **Simplify** the second expression.

$$\begin{aligned}
\frac{6}{(iy)(iy-2)} + \frac{3}{iy} &= \frac{(6)(iy) + ((iy)(iy-2))\,(3)}{((iy)(iy-2))\,(iy)} \\
&= \frac{((6) + 3(iy-2))(iy)}{(iy)^2(iy-2)} \\
&= \frac{(6 + 3(iy) - 6)(iy)}{(iy)^2(iy-2)} \\
&= \frac{3(iy)(iy)}{(iy)^2(iy-2)} \\
&= \frac{3(iy)^2}{(iy)^2(iy-2)} \\
&= \frac{3(iy)^2}{(iy-2)(iy)^2} \\
&= \frac{3}{(iy-2)}\frac{(iy)^2}{(iy)^2} \\
&= \frac{3}{(iy-2)}.
\end{aligned}$$

$$\frac{a}{b} + \frac{c}{d} = a\frac{1}{b} + c\frac{1}{d} \tag{1}$$

$$= a\frac{d}{d}\frac{1}{b} + c\frac{1}{d} \tag{2}$$

$$= ad\frac{1}{d}\frac{1}{b} + c\frac{1}{d} \tag{3}$$

$$= ad\frac{1}{b}\frac{1}{d} + c\frac{1}{d} \tag{4}$$

$$= \left(ad\frac{1}{b} + c\right)\frac{1}{d} \tag{5}$$

$$= \left(ad\frac{1}{b} + \frac{b}{b}c\right)\frac{1}{d} \tag{6}$$

$$= \left(ad\frac{1}{b} + b\frac{1}{b}c\right)\frac{1}{d} \tag{7}$$

$$= \left(ad\frac{1}{b} + bc\frac{1}{b}\right)\frac{1}{d} \tag{8}$$

$$= \left((ad + bc)\frac{1}{b}\right)\frac{1}{d} \tag{9}$$

$$= \left(\frac{ad + bc}{b}\right)\frac{1}{d} \tag{10}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}. \tag{11}$$

(1) **Division** is the same as **multiplication by the reciprocal**, here we **split**

$$\frac{a}{b} \quad \text{into} \quad a\frac{1}{b}.$$

(2) With any mathematical expression we can **multiply by a clever form of** 1, **add a clever form of** 0 or **rewrite** it in an **equivalent** way.
Here we **multiply by the clever form of one** $\frac{c}{c}$.

(3) **Division** is the same as **multiplication by the reciprocal**, here we **split**

$$\frac{d}{d} \quad \text{into} \quad d\frac{1}{d}.$$

(4) Multiplication is **commutative**, so we convert $\frac{1}{d}\frac{1}{b}$ into $\frac{1}{b}\frac{1}{d}$.

(5) In (4) we can see two **like terms** with $\frac{1}{d}$ in common, so we may **collect** them. This step is called **factoring**. The reverse operation is **distribution**.

(6) With any mathematical expression we can **multiply by a clever form of** 1, **add a clever form of** 0 or **rewrite** it in an **equivalent** way.
Here we **multiply by the clever form of one** $\frac{b}{b}$.

(7) **Division** is the same as **multiplication by the reciprocal**, here we **split**

$$\frac{b}{b} \quad \text{into} \quad b\frac{1}{b}.$$

(8) Multiplication is **commutative**, so we convert

$$\frac{1}{b}c \quad \text{into} \quad c\frac{1}{b}.$$

(9) In (8) we can see two **like terms** with $\frac{1}{b}$ in common, so we may **collect** them. This step is called **factoring**. The reverse operation is **distribution**.

(10) **Division** is the same as **multiplication by the reciprocal**, here we perform the multiplaction to **combine**

$$(ad + bc)\frac{1}{b} \quad \text{to form} \quad \frac{ad + bc}{b}.$$

(11) By **multiplying fractions** we obtain the final result.