MODEL THEORY II HOMEWORK 7

DAKOTA BLAIR

15. *Prove*

$$\operatorname{tp}(\bar{b}/A) \vdash \operatorname{tp}(\bar{b}/A\bar{c}) \Rightarrow \operatorname{tp}(\bar{c}/A) \vdash \operatorname{tp}(\bar{c}/A\bar{b}).$$

Deduce if $A \subset C$ *and* $\operatorname{tp}(\bar{b}/A) \vdash \operatorname{tp}(\bar{b}/C)$ *then* $\operatorname{tp}(\bar{c}/A) \vdash \operatorname{tp}(\bar{c}/A\bar{b})$ *for all* $\bar{c} \in C$.

Proof. Assume there is a $\bar{d}$ such that $\operatorname{tp}(\bar{d}/A) = \operatorname{tp}(\bar{c}/A), \operatorname{tp}(\bar{d}/A\bar{b}) = \operatorname{tp}(\bar{c}/A\bar{b})$ there is a $\varphi(x, \bar{a}, \bar{b}) \in \operatorname{tp}(c/A\bar{b})$ and $\mathcal{M} \models \neg\varphi(\bar{d}, \bar{a}, \bar{b})$. Then

$$(*) \qquad \{\exists y \neg\varphi(x, \bar{a}, y) \wedge \psi(y) | \psi \in \operatorname{tp}(\bar{b}/A)\} \subset \operatorname{tp}(\bar{d}/A) = \operatorname{tp}(\bar{c}/A).$$

Then in a large model we can find $\bar{b'}$ such that $\operatorname{tp}(\bar{b'}/A) = \operatorname{tp}(\bar{b}/A)$ and $\mathcal{M} \models \neg\varphi(\bar{c}, \bar{a}, \bar{b'})$ if and only if $\mathcal{M} \models \neg\varphi(\bar{d}, \bar{a}, \bar{d})$. But $\mathcal{M} \models \varphi(\bar{c}, \bar{a}, \bar{b}$, $\operatorname{tp}(\bar{b}/A) = \operatorname{tp}(\bar{b'}/A)$ and $\operatorname{tp}(\bar{b}/Ac) \neq \operatorname{tp}(\bar{b'}/A\bar{c})$ which is a contradiction.

If $\operatorname{tp}(\bar{b}/A) \vdash \operatorname{tp}(\bar{b}/C)$ then $\operatorname{tp}(\bar{b}/A) \vdash \operatorname{tp}(\bar{b}/A\bar{c})$ for all $\bar{c} \in C$ hence from above $\operatorname{tp}(\bar{c}/A) \vdash \operatorname{tp}(\bar{c}/A\bar{b})$ for all $\bar{c} \in C$.

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