PLATONIC PACKING

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Packing tetrahedrons has never been easier. Thanks to modern research it is now known that up to just over 85% of space can be filled with regular tetrahedrons, with some unpublished results reporting that packing densities of more than 85.6% are possible. This shows that when it comes to luggage, tetrahedrons beat spheres which can only fill slightly more than 74% of space at best.

An article in the December 2009 issue of the journal Nature boasts a packing density of 85.03%, the largest published value to date. This result is the combined effort of seven professors following "a conceptually different approach, using thermodynamic computer simulations that allow the system to evolve naturally towards high-density states."

A regular tetrahedron, or simply a tetrahedron, is a regular triangular pyramid, four equilateral triangles glued together to enclose a volume. You might recognize it as the shape of methane or a four-sided die used in games like Dungeons and Dragons. The packing density is the ratio of volumes of a given arrangement of tetrahedrons to an enclosing volume. Then to fill a 100 cubic centimeter cube with only tetrahedrons, water and what we know today one would still need about 15 cubic centimeters of water after packing the cube with identical tetrahedrons of any size.

Each packing has applications to high resolution imaging in microscopic medicine and materials science. In particular new metamaterials, materials made by imposing a new structure on a familiar one, are possible by taking advantage of these dense arrangements. In a twist on the usual situation, these results are direct applications of concepts from physics to mathematics. The papers are peppered with talk of quasicrystals, entropy, phase transitions and Monte Carlo simulations which are more often associated with materials science than geometry.

It may at first glance seem that stacking tetrahedrons would leave no space, resulting in a perfect 100% packing, but this belief, held even by Aristotle, is untrue. Regiomontanus revealed the gaps in tetrahedral tilings in the 15th century through a straightforward calculation correcting a mistake that lingered for close to two millennia. While this shows that no perfect tetrahedral packing exists Salvatore Torquato, a chemist at Princeton, said in a recent interview, "I'd be shocked if what we have right now is the densest. It just happens to be the densest known right now."

On the other hand in the 17th century Johannes Kepler suggested the result above, namely that the best packing of spheres resulted in a packing density just over 74%. This remained conjecture until 1998 when Thomas Hales, a mathematician at the

University of Pittsburgh, announced an incredibly intricate proof containing an unwieldy amount of computer data. His proof was finally published in the Annals of Mathematics subject to the proviso that the referees are "99% certain" of the proofs correctness. The distrust of computer aided proofs, specifically those which generate thousands of pages of material to be verified, has existed since the publication of the Four-color theorem which shared a similar controversy. This complexity is thus the impetus for a project devising a formal proof of Hales' Theorem. A formal proof accounts for each logical step from its assumptions to its conclusion. Each sentences is constructed with predetermined axioms and the rules of consequence determine the conclusion. Therefore having a formal proof is similar to having all the pieces of the puzzle, and the check is that the picture is complete. A missing piece will mar the picture and the correct conclusion cannot be reached. A formal proof for the Four-color theorem now exists which eases the controversy in the minds of many.

With the case of spheres almost certainly settled, mathematicians look to the packing of other solids. In 2006 Dr. Torquato and John Conway, a mathematician at Princeton, published results of a tetrahedral packing with a density less than 72%. Remarkably it posed the question of whether spheres pack better than tetrahedrons. However the new record and previous results due to Elizabeth Chen, a mathematics graduate student at the University of Michigan, Yang Jiao, a graduate student at Princeton, and Dr. Torquato demonstrate that tetrahedrons do pack more densely than spheres. Announcements of these and new results published this year reveal the increased interest in this problem by the mathematical community. At this point it is still unknown whether any density strictly less than 100% is possible. Determining the limit, if there is one, will take more than finding new arrangements of tetrahedrons, but will instead possibly be an undertaking comparable to settling the Kepler conjecture.